TECHNICAL NOTE

A method for determining a consistent set of radiation view factors from a set generated by a nonexact method

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This note investigates the use of radiation view factors generated by an approximate numerical method in a model of heat transfer between planar heat sources, walls fully enclosing them, and the external environment. It shows that unless two sets of compatibility conditions are satisfied by the view factors, the heat transfer from the external enclosure surfaces is not equal to that generated by the heat sources. An algorithm which ensures that the numerically generated view factors satisfy the compatibility conditions is described and an example using them demonstrates the resultant energy conservation in the system.

Keywords: radiation; radiosity; view factors; approximation

Introduction

Obtaining diffuse radiant heat flows in systems with complicated geometries is often accomplished by use of a radiation network.[1] In such a network, each surface is characterized by two nodes, with, as effective potentials, the surface blackbody emissive power node and the radiosity node (Figure 1). Each radiosity node k is linked to all other radiosity nodes j in the network, the corresponding resistance to radiation heat transfer between them being $1/A_kF_{kj}$. A_k is the surface area of the kth surface, and F_{kj} is the view factor from the kth surface to the jth surface. Each radiosity node k is also connected to its corresponding surface emissive power node by a resistance $(1-\varepsilon_k)/A_k\varepsilon_k$, where ε_k is the emissivity of surface k. The net radiant heat flow from this surface is

$$Q_k = A_k\varepsilon_k(\sigma T_k^4 - J_k)/(1-\varepsilon_k)$$

where T_k is the temperature of surface k
J_k is the radiosity associated with surface k
σ is the Stefan-Bolzmann constant

The heat exchanged between surfaces k and j is

$$Q_{kj} = A_kF_{kj}(J_k - J_j)$$

provided that the medium between them is nonabsorbing.

For given sets $\{T_k\}$, $\{F_{kj}\}$, $\{A_k\}$, and $\{\varepsilon_k\}$, the set $\{J_k\}$ needs to be found before heat fluxes can be calculated. A solution results from writing a heat balance on each radiosity node k. Referring to Figure 1 and assuming no surface can see itself, we obtain the balance

$$\frac{A_k\varepsilon_k}{1-\varepsilon_k}(\sigma T_k^4 - J_k) + \sum_{j\neq k} A_kF_{kj}(J_j - J_k) = 0$$

Expanding and dividing through by A_k gives

$$\frac{\varepsilon_k}{1-\varepsilon_k}\sigma T_k^4 - J_k\left(\frac{\varepsilon_k}{1-\varepsilon_k} + \sum_{j\neq k} F_{kj}\right) + \sum_{j\neq k} J_jF_{kj} = 0 \tag{1}$$

The sum of all view factors from one surface to the others should be unity:

$$\sum_{j\neq k} F_{kj} = 1 \tag{2}$$

Equation 1 becomes

$$J_k = \varepsilon_k\sigma T_k^4 + (1-\varepsilon_k)\sum_{j\neq k} J_jF_{kj} \tag{3}$$

An n-surface system should yield n equations of this type, allowing solution for the n radiosities. Since $\varepsilon_k \leq 1$ and $\sum_{j\neq k} F_{kj} = 1 < 1/(1-\varepsilon_k)$ the linear system of equations satisfies sufficiency conditions for the Gauss-Seidel algorithm to be used for solving it.

For systems with complicated geometries view factors need to be calculated numerically and are subject to discretization errors. In these cases, the unity summation condition and/or the reciprocal relationship

$$A_kF_{kj} = A_jF_{jk} \tag{4}$$

are not usually obeyed. In these situations, Equation 3 is not valid, and Equation 1 needs to be employed.

Enclosure heat flow incompatibilities arising from view factor errors

A model for predicting the steady-state surface temperatures inside an enclosure of zero capacitance, but containing a heat source, is given in Ref. 2. Convection and radiation to the inner

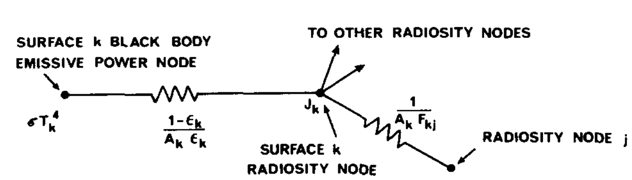

Figure 1 Radiosity network representation of node j

Table 1 Sum of heat flows through enclosure for a 600-W internal source

Case	Sum of heat flows through enclosure surfaces	Comments
A	566	Equation 2 not satisfied, Equation 4 satisfied. Equation 2 used for radiosities
B	694	Same as Case A, but Equation 1 used for radiosities
C	607	Equation 3 satisfied, but Equation 4 not satisfied. Equation 2 used for radiosities

walls, conduction through them, and then radiation and convection to the external environment were assumed to be the significant modes of heat transfer.

The radiation heat transfer was modeled by using the radiosity method[1] with view factors determined numerically from RAVFAC.[3] Heat transfer to each of the enclosure's internal surfaces was determined by summing the radiation and convection components, calculated for the steady-state surface temperatures. Because there were no numerical approximations made in the conduction and convection heat transfer models, any discrepancy between the heat generated and that flowing to the enclosure walls must be due to errors in the radiation heat transfer model. One obvious source of error in this is that the view factors do not satisfy Equations 2 and 4.

Table 1 shows the total predicted heat transfer to the walls of an enclosure containing a 600-W source. Figure 2 shows this enclosure, which has eight surfaces, including two contributed by the planar double-sided heat source. Case A corresponds to the unmodified view factors generated by RAVFAC. Equation 4 is satisfied automatically by the view factors, but Equation 2 is not. A crude facility in RAVFAC allows optional manipulation on the view factors so that Equation 2 is satisfied, but then Equation 4 is not. The result in case C was generated with these view factors, and it can be seen that energy is still not conserved. Case A shows that even when Equation 1 is used with the initially calculated view factors, an energy balance cannot be achieved.

The contour integration procedure contained in RAVFAC was used to generate the view factors. Each surface was divided into 25 subelements. Table 2 gives sums of the unmodified view factors generated at the walls, ceiling, floor, and source.

It was thought that the departure from unity of each of the sums in Table 2 caused the imbalances in Table 1. These departures are small compared with other systems modeled. In some, there were view factor sums of 1.8 and 0.6 which give rise to errors in predicted heat flows of 15%. Table 1 suggests that in order to obtain correct heat flows, the view factors used should obey Equations 2 and 4. Case C does not correspond to this situation because the normalization proceure in RAVFAC, whereby each view factor from a surface is divided by the sum

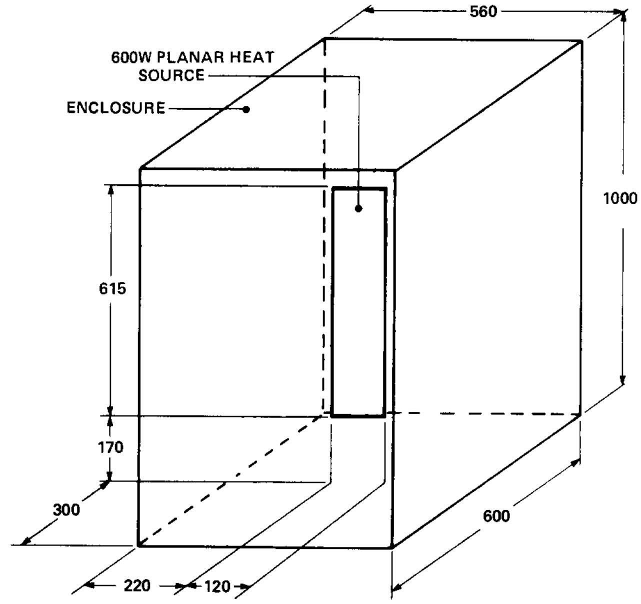

Figure 2 Details of enclosure containing 600-W planar heat source (dimensions in mm)

Table 2 Sums of view factors from each internal enclosure surface

Enclosure walls				Ceiling	Floor	Source	
0.97358	0.99711	0.97331	0.99736	0.97996	0.98086	0.99754	0.99753

of view factors between that surface and all other viewable surfaces, destroys compliance with Equation 4. For these reasons, an algorithm which forces compliance with Equations 2 and 4 needs to be applied to view factors calculated by any approximate procedure.

Decreasing the discretization error by dividing each surface into a larger number of subsurfaces certainly reduces the errors discussed above, but this always increases computation time unacceptably. RAVFAC still produced 10% of its view factor sums different from 1 by at least the amounts implied in Table 2 when each surface in a system containing complicated geometries was divided into 100 elements. Doubling the discretization in a linear dimension produces 4 times the square of the original number of elements and hence 16 times the original number of element/element subview factors which need to be calculated. The 16-fold increase in computation effort, plus the still present error described above, makes this an unacceptable solution.

An alternative procedure is to use the generated view factors $\{F_{ij}\}$ as a starting solution in a nonlinear programming problem which seeks a set of view factors $\{F'_{ij}\}$ such that $y=\sum_{i,j}(F'_{ij}-F_{ij})^2$ is minimum and the constraints imposed by Equations 3 and 4 are satisfied. However, errors associated with each F_{ij} are highly

Notation

A_k	Area of surface k, m^2
D_i	$(1-\sum_k F_{ik})/m$
F_{ij}	Radiation view factor from surface i to surface j
F'_{ij}	New estimate of F_{ij}
J_k	Radiosity associated with surface k, W m^{-2}
m	Number of non-zero-view factors from surface i (see definition of D_i)
Q	Heat flow, W
T	Temperature, K
ε	Emissivity
σ	Stefan-Boltzmann constant, 5.67×10^{-8} W m^{-2} K^{-4}

Table 3 A comparison of modified and unmodified view factors from one surface of a cubic enclosure

Unmodified	0.23911	0.16142	0.23911	0.12673	0.12585	0.081349
Modified	0.24234	0.16909	0.24229	0.13145	0.13047	0.084355

dependent on system geometry, and minimization of y requires far more computation than the algorithm presented here.

Because of discretization errors, no algorithm can produce geometrically exact view factors. However, the numerical method used should calculate view factors close to the (unknown) correct ones, and it is the aim of a corrective algorithm to produce only minor perturbations in them so that Equations 2 and 4 are satisfied. Given this, the simpler the corrective algorithm satisfying the above, the better.

Development of a simple algorithm

For each surface number i, let

$$D_i = \frac{1 - \sum_k F_{ik}}{m}$$

where m is the number of non-zero-view factors from surface number i. Then let $F'_{ik} = F_{ik} + D_i$ for each non-zero-view factor F_{ik}.

If $F'_{ik} < 0$ for any surface k, then decrease m by 1 and recalculate D_i and F'_{ik}, bypassing the view factor which made the previously calculated value of F'_{ik} negative. Repeat the procedure until all values of F'_{ik} are positive and set $F'_{ik} = F_{ik}$ for the unmodified F_{ik} (that is, those for which $F_{ik} + D_i < 0$). Finally, set

$$F'_{ki} = A_i F'_{ik} / A_k$$

Repeat the process for surface numbers $i+1, i+2, \ldots, n$, always using the most recently calculated view factors. That is, if for some i, F'_{ik} and F'_{ki} have already been calculated as described above, use F'_{ki} in place of F_{ki} in the first iteration of the above procedure for surface k. All surfaces should be processed similarly in the same order, as many times as is necessary, until all values of D_i are arbitrarily close to zero. The set of modified view factors so obtained will then satisfy Equations 3 and 4. The above procedure converges rapidly.

When the new algorithm is incorporated, the heat flow from the enclosure for which results are given in Tables 1 and 2 becomes 600.0 W; each sum of view factors is 1.0000 for all surfaces, and the reciprocal relationship is satisfied. For comparison, a selection of corresponding modified and unmodified view factors for the above simulation is given in Table 3. The remainder of the 50 nonzero modified view factors exhibit the same proximity to their unmodified counterparts as do those in Table 3. It can be seen that there are no major departures from the originally calculated set. More complicated problems show similar differences, except where the unmodified view factor sums are significantly different from 1. For these, marked differences between some modified and unmodified view factors are necessary to satisfy Equation 3.

There are other strategies which could have been adopted to produce view factors which would satisfy Equations 2 and 4, as well as being close to those originally calculated. Because of the acceptable results given by the simple algorithm described, these were not investigated.

Conclusion

In solving problems involving radiation heat transfer in an enclosure, all view factors from each surface must sum to unity, and pairs of view factors sharing common surfaces must obey the reciprocal relationship. An algorithm has been presented which makes view factors calculated by nonexact techniques satisfy these conditions, thereby ensuring energy conservation in the enclosure.

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